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ABSTRACT

Reported are the suggestions indicated by research studies to seven questions concerning what is known about the teaching and learning of mathematics. (1) What factors influence the learner's achievement? Answer: Age, intelligence, and the amount of structuring present in the materials. (2) Are there guides for motivation? Answer: Games, concrete materials, rewards, and an enthusiastic teacher seem most important. (3) Are there guides for reinforcement? Answer: Immediate knowledge of results seems most important. (4) Are there guides for retention? Answer: Meaningful developmental activities facilitate retention, followed by systematic review and practice. (5) Are there guides for transfer? Answer: The teaching must be planned with transfer as an objective; students should be taught how to transfer and generalize, preferably at their own ability level. (6) How do organization and instruction interact? Answer: No conclusion. (7) What is the role of discovery? Answer: Guided discovery groups generally achieve higher problem solving scores than expository groups; normally there is no difference on computation skill. (RS)

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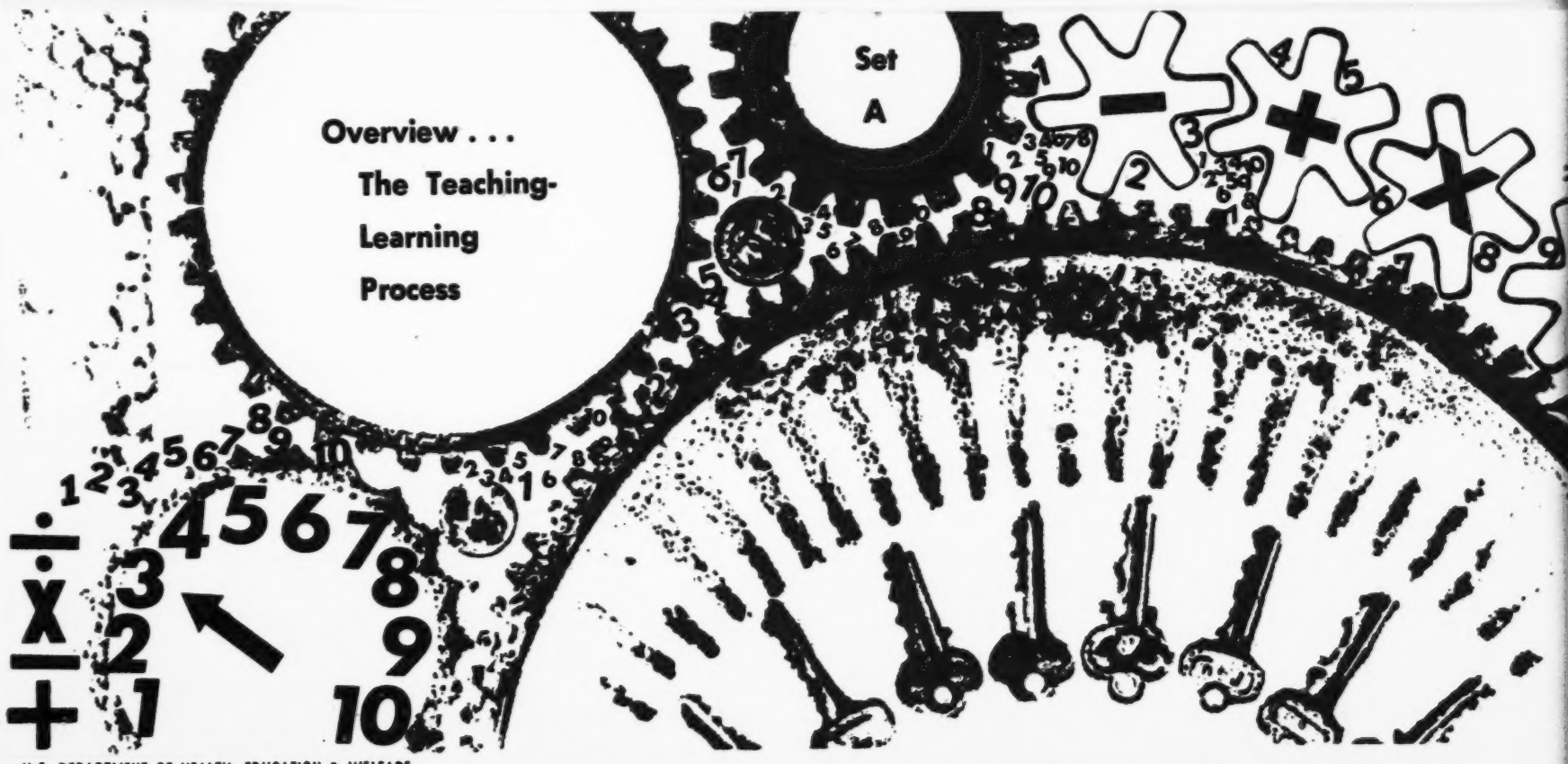

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THE TEACHING-LEARNING PROCESS

What factors associated with the learner influence achievement in mathematics?

Children appear to acquire mastery and understanding of mathematical ideas in steps or stages. Materials which are carefully structured to guide children through various "levels" tend to promote retention of the knowledge.

Age and intelligence are positively correlated with ability to learn various concepts, and thus must be considered in your planning.

Is there research to guide us in motivating learning?

It seems plausible that children must be interested in learning in order to learn. What promotes interest? Games and materials are effective; your enthusiasm and praise of their efforts are <u>essential</u>. For some children, material "rewards" may be helpful.

Is there research to guide us in reinforcing learning?

Giving children "knowledge of results," by providing scores or correct answers, seems to be one of the best ways of reinforcing their learning. Confirming a child's response is more effective than merely supplying him with the answer.

Is there research to guide us in facilitating retention?

When an experience has meaning to the learner and is understood by the learner, retention is facilitated. Planning to spend at least 50% of mathematics class time on meaningful developmental activities will help, as will allowing children to work at their own level.

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Intensive and specific review and practice should be provided, regularly and systematically, with especially careful review of material taught just before a vacation period.

Is there research to guide us in facilitating transfer?

You can help children to transfer mathematical skills and concepts from one experience to another by:

(1) planning and teaching for transfer--which implies that what is to be transferred must first be carefully determined

(2) teaching children how to transfer--which includes stress on searching for patterns and rules

(3) guiding children to generalize on the basis of experiences

(4) teaching with meaning--possibly discovery-oriented

(5) providing for instruction and practice for each child on his own level.

What is the interaction of organization and instruction variables?

The way in which the curriculum is organized--whether by areas or topics--and the way instruction is presented--either inductively or deductively--were not found to interact significantly to affect mathematical learning.

What is the role of "discovery" in the teaching-learning process?

There is much discrepancy in the way in which "discovery" is defined and used. If it is applied to a teaching approach in which the teacher leads pupils to a desired conclusion or behavior with directed questions, then it may be labelled "guided discovery." This is frequently contrasted with an "expository" approach, in which teachers explain or tell pupils what they are to do to perform a desired behavior.

When a "guided discovery" and an "expository" approach are compared, "guided discovery" groups have generally been found to achieve higher on tests of (1) retention and (2) transfer. Those taught by an "expository" approach may achieve higher scores on tests immediately following instruction.

Generally, the "guided discovery" groups achieve higher scores for problem solving than do groups taught by "exposition." However, neither approach has an advantage on measures of computational skill.

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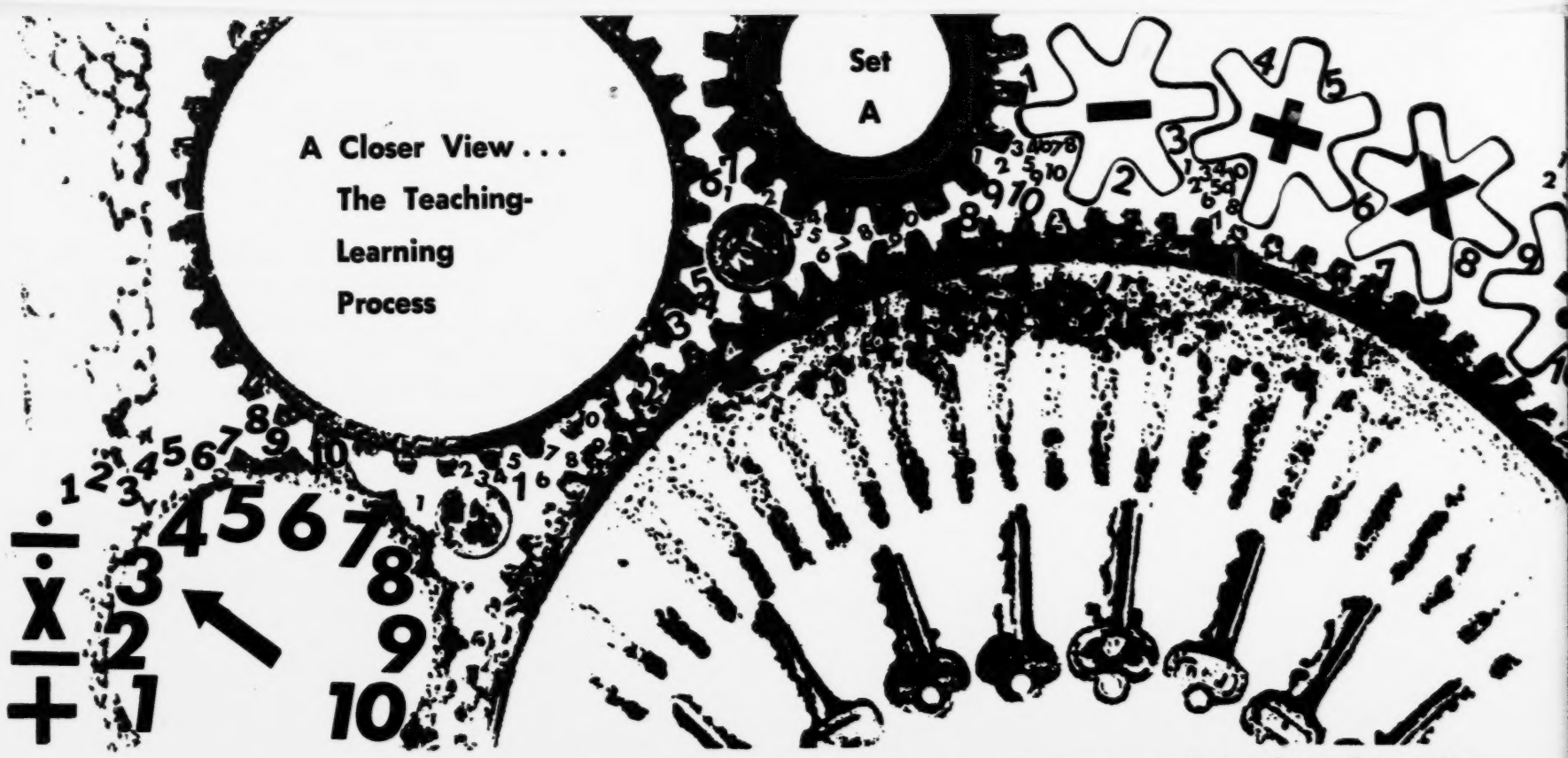
If you would like more information about the research whose findings are cited above, contact MARILYN N. SUYDAM, Project Director, at The Pennsylvania State University, University Park, Pennsylvania, 16802.



THE TEACHING-LEARNING PROCESS

Research to guide us in determining how we should teach and how children learn encompasses far more than one curriculum area. We have not attempted a broad survey of learning theory, but rather have selected that research which (1) is based on a phase of the elementary school mathematics curriculum and (2) provides specific suggestions to teachers of elementary school mathematics. Many of these findings have been substantiated not only in research across many phases of the curriculum, but also by practical use.

What factors associated with the learner influence achievement in mathematics?

Learning is not an "all or none" process. We generally acquire understanding progressively, in steps or stages. Perreault (1957) reported that the child's ability to count, to group, and to perceive the number of objects without counting appeared to reflect such developmental stages.

Gagne and Bassler (1963) structured a hierarchy of "subordinate knowledge" which led to the development of a concept. They found that, in general, sixth grade pupils learned concepts developed according to such a hierarchy. Although they did not retain all of the subordinate knowledge, they did continue to achieve well on the final task.

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It should be noted that research is variable with respect to its quality; hence, the same degree of confidence cannot be placed in all findings. An attempt has been made to take this fact into consideration in preparing this bulletin.

Brownell (1944) supplied interview data to support the conception of learning as a series of progressive reorganizations of processes and procedures. Hill (1961) found that children aged 6 through 8 could recognize the validity of logical inferences, with a pattern of steady growth rather than fixed stages.

Much additional research has shown that age and intelligence are highly related to ability to learn various specific mathematical ideas. Westbrook (1966), for instance, noted that the intellectual factors of reasoning and verbal meaning were related to achievement in mathematics in grades 4, 5, and 6. Meconi (1967) found that pupils with high ability were able to learn under any method that he investigated. Large variations in generalization ability, depending on the mathematical concept, intelligence level, and the visual pattern presented, were found on tests of varied mathematical content (Ebert, 1946).

Cathcart and Liedtke (1969) suggested that pupils in grades 2 and 3 who were identified as having a "reflective" learning style took longer to consider their responses and achieved better than pupils with an "impulsive" style. Certainly learning style needs to be considered as we plan lessons and give directions.

Is there research to guide us in motivating learning?

Exactly what "motivation" is has been the subject of some debate. Let us assume that it includes what the teacher does to increase pupils' interest in learning mathematics. (We further hope that increased interest will lead to increased achievement.) There are numerous reports about various games and materials which teachers have used successfully in increasing interest. The effect of teacher enthusiasm cannot be taken lightly.

What the teacher says--and how he says it--has been found to be particularly important. Not surprisingly, praise has been found to be a highly effective way to motivate.

Hollander (1968) recently studied the effect of different types of incentive on inner-city fifth and sixth graders following a test on addition and subtraction problems. He found that pupils worked faster when told they could earn a candy bar if they improved their own scores on a second test, and with greater accuracy when told they had performed exceptionally well. Those reproved by being told their scores were very low attempted fewer items and made more errors than were made under any of the other conditions.

Is there research to guide us in reinforcing learning?

One of the best ways of reinforcing learning is to give the child "knowledge of results"--by providing scores or by providing correct answers. Paige (1966) found that immediate reinforcement after a testing situation resulted in significantly

higher achievement scores later. Having the student respond and then giving confirmation is more effective than prompting him with the correct answer before giving him a chance to respond (McNeil, 1965.)

Kapos, Mech and Fox (1957) studied the effect of various amounts and patterns of reinforcement with third and fourth graders at several IQ levels. Different patterns of reinforcement produced differences in achievement. However, there was no clear indication of which quantity or pattern of reinforcement was best, nor was any relationship with IQ found.

Is there research to guide us in facilitating retention?

Obviously, we want children to retain what we are teaching and they are learning. There is much research to show that when something has meaning to the learner and is understood by the learner, he will be more likely to remember. Furthermore, Shuster and Pigge (1965) state that retention is better when at least 50 per cent of class time is spent on meaningful, developmental activities. Klausmeier and Check (1962) reported that when a pupil solved problems at his own level of difficulty, retention was good regardless of IQ level.

Burns (1960) reported that intensive, specific review will facilitate retention. He prepared lessons which included not only practice exercises, but also review study questions which directed pupils' attention to relevant things to consider. Meddleton (1956) pointed out that such review should be systematic.

Many teachers have noted that children fail to retain well over the summer vacation. The amount of loss varies with the child's ability and age, but how long before the vacation material was presented is especially important. Practice during the summer and review concentrated on materials presented in the spring have been shown to be especially helpful. Scott (1967) reported no systematic relationship of amount of loss and type of program, whether "traditional" or "modern."

Is there research to guide us in facilitating transfer?

Transfer infers that something learned from one experience can be applied to another experience. For instance, Olander (1931) found that pupils who studied 110 addition and subtraction combinations could give correct answers to the 90 untaught combinations. What facilitated this transfer best was instruction in generalizing, in teaching children to see patterns. Transfer increases as the similarity of problems and experiences increases. Much research has shown that meaningful instruction aids in transfer of learning. Recent studies also show that transfer is facilitated by discovery-oriented instruction.

In most studies is the implication that transfer is facilitated when teachers plan and teach for transfer--and we must teach children how to transfer. Kolb (1967), for instance,

carefully planned to have children transfer mathematical instruction to quantitative science behaviors, and achieved this transfer.

In general, the older the child and the higher his ability level, the better he can transfer. However, Klausmeier and Check (1962) found that children of various IQ levels transfer problem solving skills to new situations when the children were given work at their own level of difficulty.

What is the interaction of organization and instruction variables?

Armstrong (1968) studied the relative effects of two forms of spiral organization (area or topical) and two instructional modes of presentation (inductive or deductive). Sixth graders were assessed at each of six cognitive levels, within three areas (set theory, number theory, and geometry) and on four topics (terminology, relations, operations, and properties). The inductive mode of presentation fostered the learning of operations, while the deductive mode resulted in greater learning of mathematical properties. The interaction of curriculum organization and instructional presentation variables was not found to significantly affect mathematical learning.

What is the role of "discovery" in the teaching-learning process?

Few teachers are unaware of the word "discovery"--but there is much discrepancy about what it means as well as how it can be used. Research evidence is equivocal; perhaps the greatest factor contributing to this is the labelling of quite different methods with the same name. Nevertheless, findings from research on discovery have particular implications as we plan for the developmental aspects of mathematical teaching and learning.

In a pilot study with a small group of ten second-graders, Bassler (1968) provided groups with "intermediate guidance" in which pupils were led to a desired behavior through a "guided discovery" approach with directed questions by the teacher, or with "maximal guidance" in which teachers specifically told students what they were to do, followed by practice. The pattern of differences for posttest and retention achievement favored the "intermediate guidance" group. This group had higher transfer scores immediately following instruction, while the "maximal guidance" group had higher transfer scores on the retention test.

Fleckman (1967) reported that classes of fifth and sixth graders taught division by a "guided-discovery" method learned more concepts than classes taught by conventional textbook procedures, while computation was equivalent.

Scandura (1964) conducted several studies concerned with "exposition" versus "discovery" in classification tasks. He found that pupils taught by "discovery" were (1) better able to handle problem tasks, (2) took longer to reach the desired level of facility, and (3) seemed more self-reliant.

In an excellent study with fifth and sixth graders, Worthen (1968) compared two methods that differed in terms of sequence characteristics. In the expository method, the verbalization of the required concept or generalization was the initial step in the sequence. Mathematical principles were explained verbally and symbolically to the pupil, who then worked with examples. In the discovery method, the pupil was presented with an ordered, structured series of examples of a generalization. No explanation was given, nor any hint that there was an underlying principle to be discovered. The pupil was expected to acquire the mathematical concept or generalization through an inference of his own.

The two sequences of presentation, with carefully described teaching behaviors, resulted in significantly different pupil performance on several types of tests. In general, Worthen's findings support many of the claims made by proponents of discovery methods. The expository method was better than the discovery method on the initial test of learning, but discovery was better on retention tests administered after five and eleven weeks.

The discovery group also transferred concepts more readily and used discovery problem solving approaches to new situations better. No differences were found in pupil attitude toward the two approaches. The results further indicate that the discovery method need not take more time.

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